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SCATTERING POLARIZATION OF ARBITRARY ENVELOPES BY ANISOTROPIC STELLAR ILLUMINATION

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ABSTRACT

We model the polarization arising from electron scattering of light by a circumstellar envelope of an arbitrary shape. This is accomplished by describing the scattering function, stellar flux, and envelope density distribution in general terms using spherical harmonics and then applying their orthogonality relationships in integral expressions that describe the net observed polarization. We then take a specific example of a uniform stellar light source surrounded by an ellipsoidal shell. As an example, a polarization of 25 percent is found for the case of a disk-like star that is viewed edge-on and that is surrounded by a a disk-like envelope oriented viewed face-on to the observer.
Subject headings: Polarization, Thomson Scattering, Rayleigh Scattering

1. INTRODUCTION

Observed linear polarizations from most stars is mainly produced by scattering on circumstellar matter. There have been many papers written about the analytic treatment of polarization and scattered flux. In a seminal paper, Brown & Mclean (1977) assumed an unpolarized isotropic point source and an axisymmetric circumstellar envelope. For Thomson scattering they found that the percentage polarization was dependent upon a shape factor for the envelope, the envelope optical depth, and the inclination angle of the system in the observer;s frame. Since then many other treatments have been considered such as the generalization by Simmons (1982, 1983) for the case of Mie and Rayleigh Scattering . There has also been treatment of the case of multiple isotropic point sources (Brown, Mclean & Emslie 1978)

The past theories have one thing in common: they consider only isotropic sources. However, stars in general are anisotropic light sources. This anisotropy arises mainly through two effects: the star being non-spherical in shape or surface blemishes such as sunspots. In this paper we deal with the case of an unresolved nonspherical source, surrounded by a ellipsoidal circumstellar shell. An initial study by Al-Malki *et al.* (1999) presented results for a non-isotropic source in a spherical envelope. In Al-Malki's *et al.*, a maximum polarization was received of about 20% for a disk-like star was achieved. In this paper a somewhat higher maximum polarization of about 25% is found for a disk-like star embedded in a disk-like envelope.

In section 2 we present the general expression for the scattered flux and polarization in intergal form and define the three reference frames present within our system. We then decompose these expressions through the use of spherical harmonics. In section 3 we apply the model discussed in section 2 to the case of a small non-spherical blackbody star surrounded by an ellipsoidal shell. An approximation is considered and a maximum polarization is found for the case of a disk-like star viewed edge-on and a disk-like star view face-on. In section 4 we consider a specific example of a binary star system surrounded by an ellipsoidal shell. We conclude in section 5 with a discussion of improvements needed to make the model more complete.

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2. GENERAL EXPRESSION FOR SCATTERED FLUX AND POLARIZATION

We first need to consider the fact that there are three reference frames we will be dealing with in this paper: one for the envelope, one for the star, and one for the observer. The origin for each of these will be at the star center, hence the spherical radius r from the star center to any point in the envelope will have the same magnitude in all three frames. The following describes the three separate coordinate systems.

i) For the observer reference frame we define a cartesian coordinate system (x,y,z) centered at the star, with spherical coordinates (r,θ,ϕ), with the line-of-sight being the Oz–axis. Thus $\hat{z}$ is a unit vector in the direction of the observer from the star and x,y are observer coordinates in the plane of the sky. For a scattering point in direction $\hat{r}$, the scattering angle is given by $\cos\theta = \hat{z}\cdot\hat{r}$, as in Al-Malki *et al.* (1999). The angle $\phi = \tan^{-1} y/x$ is the observer's polarization angle (orientation) relative to Oz for any scattering point.

ii) The star's frame (X,Y,Z) with spherical coordinates (r,ϑ,φ), where OZ is a convenient stellar axis (such as a rotation axis) that lies in the $x-z$ plane of the observer's frame. The star system is rotated relative to observer coordinates (x,y,z) through standard Euler angles (α,β,γ).

iii) The envelope frame (X',Y',Z') with spherical coordinates (r,ϑ',φ'),centered on the star, where OZ' is an axis of symmetry for the enevelope.

In general the density of scatterers is $n(r,\vartheta',\varphi')$, and the flux of the radiation from the star that is taken to be unpolarized is described by $F(r,\vartheta,\varphi)$. Following Al-Malki *et al.* (1999), equation (1) gives the Stokes parameters of the scattered radiation as:

$$\left.\begin{matrix} F_{\rm sc} \\ Z^* \end{matrix}\right\} = \frac{1}{2k^2 D^2} \int\int\int n(r,\theta',\phi') F(r,\vartheta,\varphi) r^2 \times \left\{\begin{matrix} (i_1+i_2) \\ (i_1-i_2)\exp(-2i\phi) \end{matrix}\right. dr \sin\theta \, d\theta \, d\phi, \quad (1)$$

with $Z^* = Q - iU$ (where Q and U are the linear stokes parameters), $i = \sqrt{-1}$, $k = 2\pi/\lambda$ the wave number, and i_1 and i_2 the scattering functions as defined by van de Hulst (1957). For Thomson (free electrons) or Rayleigh scattering, we have

$$i_1 \pm i_2 = \frac{3k^2}{8\pi} \sigma (1 \pm \cos^2\theta), \quad (2)$$

where the value of the cross section factor σ is chosen according to whether Thomson scattering ($\sigma \propto k^2$) or Rayleigh scattering ($\sigma \propto k^4$) is being considered.

Providing that F varies smoothly, it may be conveniently expressed in terms of spherical harmonics in the observer frame (θ,ϕ),

$$F(r,\theta,\phi) = \sum_{l=0}^{\infty} \sum_{\mathrm{m}=-l}^{\mathrm{m}=l} F_{l\mathrm{m}}(r) \sum_{\mathrm{n}=-l}^{\mathrm{n}=l} R^{(l)}_{\mathrm{nm}}(\alpha,\beta,\gamma) Y_{l\mathrm{n}}(\vartheta,\varphi), \quad (3)$$

where $R^{(l)}_{\rm nm}$ are rotation matrices described by Messiah (1962). Generally, we can expand the density distribution of scatterers in terms of spherical harmonics, which for the observer's frame becomes

$$n(r,\theta,\phi) = \sum_{l'=0}^{\infty} \sum_{\mathrm{m}'=-l'}^{\mathrm{m}'=l'} n_{l'\mathrm{m}'}(r) Y_{l'\mathrm{m}'}(\theta,\phi). \quad (4)$$

Using the properties of the product of two spherical harmonics (Messiah 1962), we can express the multipoles of $n(r,\theta,\phi)$ and $F(r,\theta,\phi)$ as:

$$n(r,\theta,\phi) F(r,\theta,\phi) = \sum_{lmn} R^l_{\mathrm{nm}} \sum_{l'\mathrm{m}'} C^{\mathrm{LM}}_{ll'\,\mathrm{nm}'} Y_{\mathrm{LM}}(\theta,\phi). \quad (5)$$

In this past expression, the factors $C^{\rm LM}_{ll'\,\rm nm'}$ are Clebsh-Gordon coefficients, arising from the products of two spherical harmonics. Only terms satisfying the following two conditions contribute to the sum in Equation 5:

$$\mathrm{n} + \mathrm{m}' = M, \quad (6)$$

and,

$$|l - l'| \le l \le l + l'. \quad (7)$$

The coefficients are given, using Racah notation, by

$$C^{\mathrm{LM}}_{ll'\,\mathrm{nm}'} = (-1)^{\mathrm{M}} \sqrt{\frac{(2l+1)(2l'+1)(2L+1)}{4\pi}} \times \begin{pmatrix} l\ l'\ L \\ 0\ 0\ 0 \end{pmatrix} \begin{pmatrix} l & l' & L \\ n & m' & M \end{pmatrix} \quad (8)$$

(c.f., Messiah 1962, where values of the Clebsh-Gordon coefficients and the rotation martices are tabulated).

In order to exploit the orthogonality properties of spherical harmonics we must express the factors in the scattering function also in terms of spherical harmonics.

$$(i_1+i_2) \propto 1 + \cos^2\theta = \frac{4}{3} \left[\sqrt{4\pi} Y_{00} + \frac{\pi}{5} Y_{20}(\theta) \right], \quad (9)$$

and

$$(i_1+i_2) \propto \sin^2\theta \exp(-2i\phi) = 4 \frac{\sqrt{2\pi}}{15} Y^*_{22}(\theta,\phi), \quad (10)$$

So once substituted into the integrals contained in Equation 1, together with the other expressions above, and using the properties of spherical harmonics, $F_{\rm sc}$ and Z^* can be rewritten as:

$$F_{\rm sc} = \frac{\sigma}{4\pi D^2} \sum_{l\mathrm{mn}} R^l_{\mathrm{nm}}(\alpha,\beta,\gamma) \times \left[\sqrt{4\pi} C^{00}_{ll'nm'} + \sqrt{\frac{\pi}{5}} C^{20}_{ll'nm'} \right] S_{ll'mm'}, \quad (11)$$

and

$$R^l_{\mathrm{nm}}(\alpha,\beta,\gamma) \sum_{l'\mathrm{m}'} C^{22}_{ll'nm'} S_{ll'mm'}, \quad (12)$$

where

$$S_{ll'mm'} = \int_0^\infty F_{l\mathrm{m}}(r) n_{l'\mathrm{m}'}(r) r^2 \, dr, \quad (13)$$

with

$$F_{l\mathrm{m}}(r) = \int_{-1}^{1} \int_0^{2\pi} F(r,\theta,\phi) Y^*_{l\mathrm{m}}(\theta,\phi) \, d(\cos\theta) \, d\phi, \quad (14)$$

and

$$n_{l'\mathrm{m}'}(r) = \int_{-1}^{1} \int_0^{2\pi} n(r,\theta',\phi') Y^*_{l'\mathrm{m}'}(\theta',\phi') \, d(\cos\theta') \, d\phi'. \quad (15)$$

Equations 14 and 15 describe the effects of each function (F and n) in the appropriate stellar or envelope frame (c.f., Simmons 1982).

If the functions are smooth, the summations will converge rapidly, so the first few terms with l and $l' \le 2$ will give reasonable approximations. Due to the conditions of Equations 6 and 7, the summation over l, l', m, and m' in Equations 11 and 12 will be limited. For functions describing the stellar flux and the circumstellar density distribution of scattering particles symmetrically about stellar and envelope polar axes, respectively, the values of $S_{ll'\mathrm{mm}'}$ will be zero for odd values of l and l'.

For a spherical envelope with $n(r,\theta,\phi) = n(r)$ and an anisotropic light source, Equations 11 and 12 reduce to the forms discussed in Al-Malki et al. (1999). On the other hand, for an isotropic light source within an axisymmetric enevlope, the expressions of Brown & McLean (1977) and Simmons (1982) are recovered.

3. ELLIPSOIDAL LIGHT SOURCE AND ELLIPSOIDAL CIRCUMSTELLAR ENVELOPE

As an illustration of the preceding general formulation, we will take the case of a small blackbody star of uniform surface temperature embedded within an ellipsoidally shaped circumstellar envelope. For such a star of isotropic surface intensity I_*, the flux $F(r,\vartheta,\varphi)$ can be expressed in terms of the projected area $A_{\rm p}(\vartheta,\varphi)$ of the star as seen from direction (ϑ,φ). The flux is given by

$$F_*(r,\vartheta,\varphi) \approx I_* \, \Delta\Omega = I_* \frac{A_{\rm p}(\vartheta,\varphi)}{r^2}, \quad (16)$$

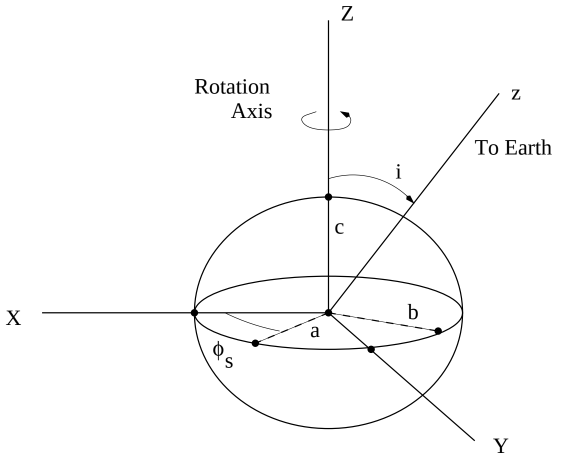

FIG. 1.— The ellipsoidal star coordinate system, where the c-axis is along the Z-axis (i.e., the rotation axis.)

where I_* is the isotropic intensity of the stellar surface and $\Delta\Omega$ is the solid angle subtended by the scattering element located a distance r from the star. For this star we define axes (a,b,c) to be along (X,Y,Z) (see Figure 1). The $A_{\rm p}$ function is defined by the following:

$$A_{\rm p} = \pi \left| \sqrt{(bc\lambda)^2 + (ac\mu)^2 + (ab\nu)^2} \right|, \tag{17}$$

where $(\lambda,\mu,\nu) = (\cos\varphi\sin\vartheta, \sin\varphi\sin\vartheta, \cos\vartheta)$ are the (X,Y,Z) direction cosines. So we obtain finally,

$$F_{l{\rm m}}(r) = \frac{I_*}{r^2} \int_0^{\pi} \int_0^{2\pi} A_{\rm p}(\vartheta,\varphi) Y^*_{l{\rm m}}(\vartheta,\varphi) \sin\vartheta \, d\vartheta \, d\varphi. \tag{18}$$

We take the circumstellar envelope to be the same as that of Simmons (1982), an ellipsoidal shell of arbitrary thickness and uniform density. The density distribution, which has an axis of rotational symmetry OZ', has an inclination angle $i_{\rm e}$ with the line of sight. We shall consider only the case of rotation of OZ' about the line of sight, but not any other direction, with an azimuthal angle $\phi_{\rm e}$ in the observer frame (see Figure 2). We then have (Simmons 1982):

$$n_{l'{\rm m}'}(r) = 2\pi (R_1 - R_2) n_0 K_{l'} Y_{l'{\rm m}'}(i_{\rm e}, \phi_{\rm e}). \tag{19}$$

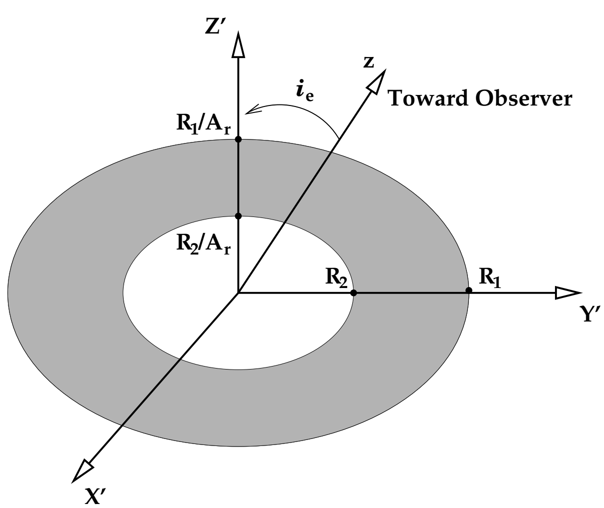

FIG. 2.— Definitions of R_1, and R_2 . This figure further defines the coordinate system and geometry of the ellipsoidal shell.

The outer and inner radii of the shell are, respectively,

$$r_{1,2}(\mu) = \frac{R_{1,2}}{\sqrt{1 + (A_{\rm r}^2 - 1)\mu^2}}, \tag{20}$$

where

$$\mu = \cos\zeta. \tag{21}$$

The lengths R_1 and R_2 are the outer and inner equatorial axis length, and $A_{\rm r}$ is the ratio of the length of the equatorial axis to the polar axis (see Figure 2). The angle ζ is the angle between the radius vector and the axis of symmetry, OZ', which is related to our frames by the addition theorem of spherical harmonics. (This explains the appearance of $Y_{l'{\rm m}'}(i_{\rm e},\phi_{\rm e})$ in Equation 19 – see Simmons 1982 and Jackson 1975.) Finally, $K_{l'}$ values represent moments of the envelope. They can be related to the shape factor found in Brown & Mclean (1977) and are given by :

$$K_{l'} = \int_{-1}^{1} \frac{P_{l'}(\mu)}{\sqrt{1 + (A_{\rm r}^2 - 1)\mu^2}} d\mu. \tag{22}$$

Inserting equations (18) to (22) in Eq. (13), we obtain

$$S_{ll'{\rm mm}'} = I_* N f_{l{\rm m}} K_{l'} Y_{ll'{\rm m}'}(i_{\rm e}, \phi_{\rm e}), \tag{23}$$

where we have defined a column density

$$N = 2\pi (R_1 - R_2) n_0, \tag{24}$$

and

$$f_{l{\rm m}} = \int_0^{\pi} \int_0^{2\pi} A_{\rm p}(\vartheta,\varphi) Y^*_{l{\rm m}}(\vartheta,\varphi) \sin\vartheta \, d\vartheta \, d\varphi. \tag{25}$$

Note that the spherical harmonic decomposition coefficients of the flux $f_{l{\rm m}}$ are now functions of the star's shape and size only (i.e., of a, b, and c). Moreover, the spherical harmonic decomposition coefficients of the envelope $K_{l'}$ are also functions of the envelope's shape and size only (i.e., of $A_{\rm r}$). This combination of $K_{l'}$ and $f_{l{\rm m}}$ determines the polarization and the scattered flux, depending on whether the two functions enhance or offset one another.

If we describe the orientation of the star relative to the observer frame in terms of the Euler angles, then we can choose α as zero, $\beta = i_s$ is the viewing inclination of the OZ–axis (i.e., the rotation axis of the star) to the line of sight Oz–axis, and $\gamma = \phi_s$ as the azimuth of the OZ–axis from the Ox–axis as measured about the Oz–axis. Thus $\phi_{\rm s}$ measures the rotational position or phase of the star relative to the observer (see Figure 3).

For the observational situation, the normalized scattered flux and Stokes parameters are usually used. They are given by $(f_{\rm sc}, q, u) = (F_{\rm sc}, Q, U)/F_{\rm tot}$, where the total flux received $F_{\rm tot}$ comprises the combination of the scattered flux $F_{\rm sc}$ plus direct flux from the star. This contribution by direct stellar light we denote as $F_* = I_* A_{\rm p}(i_{\rm s}, \phi_{\rm s})/D^2$. So the total observed flux becomes

$$F_{\rm tot} = \frac{I_*}{D^2} \left[A_{\rm p}(i_{\rm s}, \phi_{\rm s}) + (D^2/I_*) F_{\rm sc} \right]. \tag{26}$$

The general expressions for the normalized scattered flux and stokes parameters become

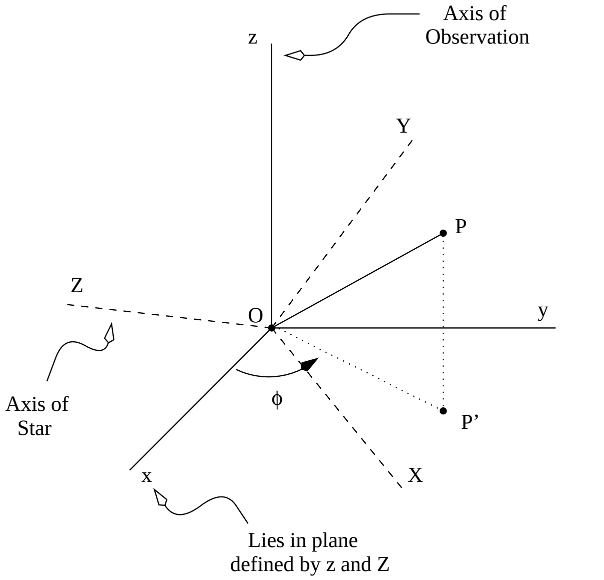

FIG. 3.— Definitions of the star's and observers coordinate systems. The point O marks the emitting anisotropic star (point source), and P a general scattering particle in the envelope. OZ is the rotation axis, where θ is the scattering angle, and ϕ is the polarization direction. The Euler angles are $\alpha = 0$, $\beta = i_s$ the inclination angle between Oz and OZ, and $\gamma = \phi_s$ the angle from the X-axis in the xy-plane. The scattering plane is $P'OP$.

$$f_{\rm sc} = \frac{\tau}{4\pi F_{\rm tot}} \sum_{l{\rm mn}} R^l_{\rm nm}(0, i_{\rm s}, \phi_{\rm s})\, f_{l{\rm m}} \times \left[\sqrt{4\pi} C^{00}_{ll'{\rm nm}'} + \sqrt{\frac{\pi}{5}} C^{20}_{ll'{\rm nm}'}\right] \quad (27)$$

and with $z^* = q - {\rm i}u$,

$$z^* = \frac{3\tau}{4\pi F_{\rm tot}} \sqrt{\frac{2\pi}{15}} \sum_{l{\rm mn}} R^l_{\rm nm}(0, i_{\rm s}, \phi_{\rm s})\, f_{l{\rm m}} \sum_{l'{\rm m}'} K_{l'}\, Y_{l'{\rm m}}(i_{\rm e}, \phi_{\rm e}) C^{22}_{ll'nm'}\, S_{ll'mm'}, \quad (28)$$

where τ is the average envelope optical depth, equal to $\sigma(R_1 - R_2)n_0$ (see Simmons 1982). The degree of polarization is given by $p = |z^*| = |z| = \sqrt{q^2 + u^2}$, and the polarization direction is given by $\phi_p = \frac{1}{2}\tan^{-1}(u/q)$.

From Equations 27 and 28, we can calculate the polarization and the scattered flux, using the properties of the two factors $K_{l'}$ and $f_{l{\rm m}}$, that describe the envelope and stellar anisotropy. The stellar flux and the scatterer density functions, $K_{l'}$ and $f_{l{\rm m}}$ are non-zero only for even values of l' and l.

4. APPLICATIONS

We now examine the theory in more detail and create an approximation that can be applied to many cases. As previously noted, the multipoles of the flux $f_{l{\rm m}}$ are non-zero only for even l, and the spherical harmonics for $l \geq 4$ are important only for fairly large stellar distortions from sphericity. For most values of c, f_{20} domintes f_{l0} for $l \geq 2$. We therefore neglect the effects of $f_{l{\rm m}}$ for $l > 2$.

For $l = l' = 2$, we can write the scattered flux as

$$\begin{aligned} F_{\rm sc} = \frac{\sigma}{4\pi D^2} \Bigg\{ \sqrt{4\pi} \Bigg[& \sum_{\rm m=-2}^{\rm m=2} S^{22\,2-2}_{\rm m} C^{00}_{22\,2-2} \\ & + \sum_{\rm m=-2}^{\rm m=2} S^{22\,1-1}_{\rm m} C^{00}_{22\,1-1} + \sum_{\rm m=-2}^{\rm m=2} S^{22\,00}_{\rm m} C^{00}_{22\,00} \\ & + \sum_{\rm m=-2}^{\rm m=2} S^{22\,-11}_{\rm m} C^{00}_{22\,-11} + \sum_{\rm m=-2}^{\rm m=2} S^{22\,-22}_{\rm m} C^{00}_{22\,-22} \\ & + S^{00\,00}_{\rm m} C^{00}_{00\,00} \Bigg] + \sqrt{\frac{\pi}{5}} \Bigg[S^{02\,00}_0 C^{20}_{02\,00} \\ & + \sum_{\rm m=-2}^{\rm m=2} S^{22\,2-2}_{\rm m} C^{20}_{22\,2-2} + \sum_{\rm m=-2}^{\rm m=2} S^{22\,1-1}_{\rm m} C^{20}_{22\,1-1} \\ & + \sum_{\rm m=-2}^{\rm m=2} S^{22\,00}_{\rm m} C^{20}_{22\,00} + \sum_{\rm m=-2}^{\rm m=2} S^{22\,-11}_{\rm m} C^{20}_{22\,-11} \\ & + \sum_{\rm m=-2}^{\rm m=2} S^{22\,-22}_{\rm m} C^{20}_{22\,-22} + \sum_{\rm m=-2}^{\rm m=2} S^{20\,00}_{\rm m} C^{20}_{20\,00} \Bigg] \Bigg\}, \end{aligned} \quad (29)$$

and the Stokes parameters as

$$\begin{aligned} z^* = \frac{3\sigma}{4\pi D^2} \sqrt{\frac{2\pi}{15}} \Big\{ & S^{02\,02}_0 C^{22}_{02\,02} \\ & + \sum_{\rm m=-2}^{\rm m=2} S^{22\,20}_{\rm m} C^{22}_{22\,20} + \sum_{\rm m=-2}^{\rm m=2} S^{22\,11}_{\rm m} C^{22}_{22\,11} \\ & + \sum_{\rm m=-2}^{\rm m=2} S^{22\,02}_{\rm m} C^{22}_{22\,02} + \sum_{\rm m=-2}^{\rm m=2} S^{20\,20}_{\rm m} C^{22}_{20\,20} \Big\}, \end{aligned} \quad (30)$$

where

$$S^{ll'\,{\rm nm}'}_{\rm m} = R^l_{\rm nm}(\alpha, \beta, \gamma) S_{ll'\,{\rm mm}'}. \quad (31)$$

In the case of a spherical envelope and a disk-like star ($a = b = 1, c = 0$) , a polarization of p20% is theoretically achievable (Al-Malki *et al.* 1999). Our more general model does reproduce this value for a spherical envelope. As an extreme case to investigate large polarization values, we use the disk star example of Al-Malki *et al.* but now with a disk-shaped envelope. A maximu polarization of about 25 % is produced for the disk-shaped star ($a = b = 1$ and $c = 0$) viewed edge-on at $i_{\rm s} = 90°$ with disk-shaped envelope that is oriented face-on to the observer ($i_{\rm e} = 0°$) and orthogonal to the star. Note that as long as the envelope is optically thin, the quoted polarization values for these extreme cases are independent of the envelope optical depth τ.

We have considered applications of equations (29) and (30) to a single fast rotating oblate star, a Roche lobe filling star, and a non-radially oscillating star. The results yield qu–diagrams of similar shape as in the spherical envelope case of Al-Malki *et al.* (1999), but with smaller or larger values of q and u. The main difference is that the two functions of F and n for the star flux and envelope density distribution can enhance or offset each other. These models will be discussed in a separate paper.

5. CONCLUSION

In this work a model for the polarization arising from an anisotropic point light source within an arbitraryily shaped envelope has been presented. We specifically considered Thomson and Rayleigh scattering mechanisms. The mathematical analysis made full use of spherical harmonics decomposition and their properties. By using the first few spherical harmonics, an acceptable approximation was derived to represent star and envelope distortions by substantial amounts. The application of the general model to a disk-like star with zero thickness viewed edge-on surrounded by a disk-like envelope oriented pole-on to the observer gave a polarization of 25%.

In the future we would like the effects of the finite star size, effects that tend to lower the net polarization (Cassinelli, Nordsieck, & Murison 1987). We have started to include these effects in the specific case of a cool, evolved star to model a laterally structured chromosopheric envelope (Ignace, Henson, & Carson 2007), but the approach has not yet been generalized.

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